# The Structure of Dicalcium Potassium Heptahydrogen Tetrakis(phosphate) Dihydrate, $\mathbf{C a}_{2} \mathrm{KH}_{\mathbf{7}}\left(\mathrm{PO}_{4}\right)_{4} \cdot \mathbf{2} \mathrm{H}_{2} \mathbf{O}$, by X-ray and Neutron Diffraction 

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#### Abstract

M_{r}=542.25\), triclinic, $a=5.676$ (1), $b=$ 12.210 (2), $\quad c=6.292$ (1) $\AA, \quad \alpha=104.10$ (3),$\quad \beta=$ $115 \cdot 16$ (2), $\gamma=84.25(2)^{\circ}, V=382.8$ (6) $\AA^{3}, Z=1$, $D_{x}=2.352 \mathrm{Mg} \mathrm{m}^{-3}, \quad \mu=1.51 \mathrm{~mm}^{-1}, \quad F(000)=274$ (X-rays) $, \quad \mu=0.14 \mathrm{~mm}^{-1}, \quad F(000)=96.5 \mathrm{fm}$ (neutrons), $T=298 \mathrm{~K}$. For X-rays, space group $P \overline{1}$, Mo $K \alpha, \lambda=0.7107 \AA, R=0.024$ for 2040 independent observed reflections. For neutrons, space group $P 1, \lambda=1.273 \AA, R=0.051$ for 1383 independent observed reflections. The structure is isomorphous with $\mathrm{Ca}_{2}\left(\mathrm{NH}_{4}\right) \mathrm{H}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$. Although K could occupy a center of symmetry, it apparently does not, and two of the three hydrogen bonds that would cross centers of symmetry in $P \overline{1}$ are also markedly asymmetric. Because of the greater sensitivity of neutrons to hydrogen atoms, it is concluded that the correct space group is $P 1$.


Introduction. In a study of the compound $\mathrm{Ca}_{2}\left(\mathrm{NH}_{4}\right)$ $\mathrm{H}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$ (Takagi, Mathew \& Brown, 1980) it was found that the structure could be successfully refined in space group $P \overline{1}$, although this required the ammonium ion to be statistically disordered in the vicinity of a center of symmetry. However, the resulting structure contained three hydrogen bonds that crossed centers of symmetry that were unusually long ( $\geq 2.50 \AA$ ) for symmetric hydrogen bonds. It was not possible using X -ray diffraction to study the hydrogen positions to determine whether the bonds were truly symmetric, were disordered across the centers of symmetry, or whether the structure was actually noncentrosymmetric. Single crystals of the ammonium compound large enough for neutron diffraction measurements were not available, but some large single crystals of an isomorphous compound with potassium substituted for the ammonium were obtained. This study was undertaken to determine the symmetry of the compound and the configuration of the hydrogen bonds.

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Experimental. Crystals prepared by procedure of Flatt, Brunisholz \& Chapuis-Gottreux (1951). Cell parameters by least-squares fit of $2 \theta$ values of 15 reflections in the range $48-54^{\circ}$, measured at both positive and negative settings on a Picker X-ray diffractometer using graphite-monochromated Mo $K \alpha_{1}$ radiation ( $\lambda=0.70932 \AA$ ). X-ray diffraction data collected using $\theta-2 \theta$ scan technique. Diffractometer control programs of Lenhert (1975). Neutron diffraction data collected on four-circle neutron diffractometer at NBS research reactor using a procedure described previously (Prince, 1972), in which a preliminary measurement of peak height is made, and only those reflections whose peak heights exceed background by at least $2 \sigma$ are considered to be observed and measured further by a $\theta-2 \theta$ scan. The cell parameters from the X-ray measurements were used. Crystal and experimental data are given in Table 1. Intensities reduced to structure factors, structure refined using RFINE 4 (Finger \& Prince, 1975). $\sum w_{i}\left(\left|F_{o i}\right|-\right.$ $\left.\left|F_{c i}\right|\right)^{2}$ minimized. Scattering factors and anomalous dispersion corrections for X-rays and scattering factors for neutrons from International Tablesfor $X$-ray Crystallography (1974). Patterson map and electron density maps confirmed that the structures of the potassium and ammonium salts were isomorphous. Therefore, atomic coordinates of $\mathrm{Ca}_{2}\left(\mathrm{NH}_{4}\right) \mathrm{H}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$ (Takagi et al., 1980) were used as initial parameters in refinement of X-ray data. Neutron refinement weights determined initially by $w_{i}=1 /\left[\sigma_{c i}^{2}+\left(0.025 F_{o i}\right)^{2}\right]$ where $\sigma_{c i}^{2}$ is variance of $i$ th structure factor based on counting statistics. In later stages of refinement robust/resistant weighting scheme used (Nicholson, Prince, Buchanan \& Tucker, 1983): $w_{i}^{\prime}=w_{i}\left[1-\left(r_{i} / s\right)^{2}\right]^{2}$ for $\left|r_{i} / s\right| \leq 1$, $w_{i}^{\prime}=0$ otherwise. $r_{i}$ denotes $w_{i}^{1 / 2}\left(\left|F_{o i}\right|-\left|F_{c i}\right|\right), s$ is resistant measure of scale chosen to be $9\left|r_{i}\right|_{m}$, where $\left|r_{i}\right|_{m}$ is median of absolute values of $r_{i}$. Initial parameters from preliminary refinement of X-ray data. Four different versions of model used: (1) space group (c) 1984 International Union of Crystallography

Table 1. Crystal, experimental and final refinement data for $\mathrm{Ca}_{2} \mathrm{KH}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$

|  | X-ray | Neutron |
| :--- | :---: | :---: |
| Crystal dimensions (mm) | $0.23 \times 0.22 \times 0.09$ | $5.0 \times 3.0 \times 1.0$ |
| Wavelength $(\AA)$ | 0.7107 | 1.273 |
| $2 \theta$ max $\left(^{\circ}\right)$ | 60 | 116 |
| Scan rate | $0.5^{\circ} \mathrm{min}^{-1}$ | Variable (see text) |
| Number of unique |  |  |
| reflections | 2145 | 1872 |
| Number of observable |  |  |
| $\quad$ reflections | $2040\left[>3 \sigma\left(F_{o}\right)\right]^{*}$ | 1398 |
| Range of $h, k, l$ | $0-7,-16-17,-7-7$ | $0-7,-15-16,-8-7$ |
| Absorption correction |  |  |
| $\quad$ range | $1.13-1.34 \dagger$ |  |
| Space group used | $P \mathrm{I}$ |  |
| Number of variables | 138 | $P 1$ |
| $R$ | 0.024 | $0.051(1383$ reflections) |
| $w R$ | 0.042 | 0.060 |
| $w^{-1}$ | $\sigma^{2}\left(F_{o}\right)+\left(0.02 F_{o}\right)^{2}$ | See text |
| $S$ | 1.77 | 2.15 |
| Max. $\Delta / \sigma$ | 1.2 | 0.10 |
| Av. $\Delta / \sigma$ | 0.1 | 0.03 |
| $\Delta \rho$ | $<0.4 \mathrm{e} \AA^{-3}$ | $0.02 \ddagger$ |
| Extinction parameter $\S$ | $2.7(4) \times 10^{-5}$ | $9(3) \times 10^{-7}$ |

* Calculated from counting statistics.
$\dagger$ The absorption correction was applied using XRAY76 (Stewart, Machin, Dickinson, Ammon, Heck \& Flack, 1976). For neutrons no absorption corrections were applied.
$\ddagger$ The unit is density of nuclei with scattering length 10 fm .
§ Zachariasen (1967). For neutrons the parameter is the product $r \times \bar{t}$.
$P \overline{1}$, with potassium atom on center of symmetry; (2) space group $P \overline{1}$, with half a potassium atom in each of two positions symmetrically displaced from center; (3) space group $P 1$, with potassium atom in single off-center position and with temperature factors of pairs of atoms related by center in models 1 and 2 constrained to be equal; and (4) space group $P 1$, with all temperature factors unconstrained. Because of well known numerical problems with refinement in noncentrosymmetric space groups of structures that are almost centrosymmetric first few cycles of model 3 utilized an alternative more stable algorithm of the type known as a quasi-Newton algorithm (Broyden, 1972). Table 2 is a summary of the $R$ indices and numbers of parameters for all four models.

Discussion. The final atomic parameters for the X-ray and neutron refinements are listed in Tables 3 and 4, respectively. Inasmuch as the structures of $\mathrm{Ca}_{2}\left(\mathrm{NH}_{4}\right)$ $\mathrm{H}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$ and $\mathrm{Ca}_{2} \mathrm{KH}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$ are isomorphous, and the structure of the former has been described in detail (Takagi et al., 1980), the discussion will be limited to the bonds involving hydrogen atoms, as determined in the neutron refinement.

As can be seen in Table 2, each of the three lower symmetry models shows an improvement in the fit, relative to the next more restrictive model. In each case the $R$ ratio test (Hamilton, 1965) shows that the probability of observing that improvement by chance is essentially zero (less than $10^{-6}$ ). Model 4, however, has

Table 2. Summary of the $R$ indices in the neutron diffraction refinement, the number of reflections included in the refinement and the number of parameters for each of four models of $\mathrm{Ca}_{2} \mathrm{KH}_{7}{ }^{-}$
$\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$

| $\quad$ Model | Number of <br> reflections | Number of <br> parameters | $R(\%)$ | $R_{\kappa} \cdot(\%)$ |
| :--- | :---: | :---: | :---: | :---: |
| (1) $P \overline{1}-K$ on center | 1381 | 170 | 5.9 | 7.0 |
| (2) $P \overline{1}-K$ split | 1381 | 173 | 5.7 | 6.7 |
| (3) $P 1-T F$ 's constrained | 1383 | 227 | $5 \cdot 1$ | 6.0 |
| (4) $P$ 1-unconstrained | 1379 | 323 | 4.5 | $5 \cdot 2$ |

Table 3. Refined position parameters and equivalent isotropic thermal parameters for the $X$-ray refinement of $\mathrm{Ca}_{2} \mathrm{KH}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$
E.s.d.'s for the least significant digits are given in parentheses.

|  | $x$ | $y$ | $z$ | $B_{\text {eq }}^{*}\left(\AA^{2}\right)$ |
| :--- | :---: | :---: | :---: | :---: |
| Ca | $-0.00076(5)$ | $0.09902(2)$ | $0.29829(5)$ | $0.90(2)$ |
| $\mathrm{K} \dagger$ | $0.49694(28)$ | $0.53752(14)$ | $-0.01372(30)$ | $4.84(8)$ |
| $\mathrm{P}(1)$ | $0.15159(8)$ | $0.63698(3)$ | $0.37840(8)$ | $1.19(2)$ |
| $\mathrm{P}(2)$ | $0.49717(7)$ | $-0.09738(3)$ | $0.21138(6)$ | $0.84(2)$ |
| $\mathrm{O}(11)$ | $0.1048(3)$ | $0.5952(1)$ | $0.1098(2)$ | $2.02(7)$ |
| $\mathrm{O}(12)$ | $0.1388(2)$ | $0.7647(1)$ | $0.4351(2)$ | $1.27(5)$ |
| $\mathrm{O}(13)$ | $0.4220(3)$ | $0.5984(1)$ | $0.5390(3)$ | $2.25(6)$ |
| $\mathrm{O}(14)$ | $-0.0615(3)$ | $0.5853(1)$ | $0.4134(3)$ | $2.04(7)$ |
| $\mathrm{O}(21)$ | $0.2440(2)$ | $-0.0352(1)$ | $0.1426(2)$ | $1.25(6)$ |
| $\mathrm{O}(22)$ | $0.7464(2)$ | $-0.0320(1)$ | $0.3192(2)$ | $1.21(5)$ |
| $\mathrm{O}(23)$ | $0.5312(2)$ | $-0.1685(1)$ | $0.3986(2)$ | $1.46(6)$ |
| $\mathrm{O}(24)$ | $0.4632(3)$ | $-0.1833(1)$ | $-0.0326(2)$ | $1.84(6)$ |
| $\mathrm{O}(3)$ | $0.1434(3)$ | $0.2477(1)$ | $0.1743(2)$ | $1.55(6)$ |
| $\mathrm{H}(23)$ | $0.392(5)$ | $-0.191(2)$ | $0.396(4)$ | $2.8(5)$ |
| $\mathrm{H}(24)$ | $0.570(6)$ | $-0.205(3)$ | $-0.059(5)$ | $3.4(7)$ |
| $\mathrm{H}(31)$ | $0.034(5)$ | $0.284(2)$ | $0.052(4)$ | $2.5(5)$ |
| $\mathrm{H}(32)$ | $0.259(6)$ | $0.303(3)$ | $0.299(6)$ | $5.2(7)$ |
| $\mathrm{H}(11)$ | 0.0 | 0.5 | 0.0 | $8.6(13)$ |
| $\mathrm{H}(13)$ | 0.5 | 0.5 | 0.5 | $13.2(15)$ |
| $\mathrm{H}(14)$ | 0.0 | 0.5 | 0.5 | $3.4(8)$ |

* Hamilton (1959).
$\dagger$ Atom is statistically disordered across a center of symmetry. An occupancy factor of 0.5 was used in all calculations.
one anisotropic temperature factor matrix that is not positive definite, and the refined position parameters are not markedly different from those of model 3. For this reason further discussion of the detailed structure will be based on the parameters of model 3. Table 5 gives a list of selected bond distances for both refinements,* and Table 6 gives details of hydrogenbond configurations. Fig. 1 is an overall view of the structure.

The immediately apparent salient feature of model 1 was a very large anisotropy in the temperature factor of the potassium atom, and the fact that there was substantial improvement in the fit in going to model 2 with only three additional parameters is a strong

[^0]Table 4. Refined position parameters and equivalent isotropic thermal parameters for the neutron refinement

$$
\text { of } \mathrm{Ca}_{2} \mathrm{KH}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}
$$

E.s.d.'s for the least significant digits are given in parentheses. K was held fixed to define the origin.

|  | $x$ | $y$ | $z$ | $B_{\text {eq }}^{*}\left(\AA^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| Ca | $-0.0032(21)$ | 0.0991 (12) | $0 \cdot 3005$ (20) | 0.87 (3) |
| Ca | -0.0019 (21) | -0.0980 (12) | -0.2961 (21) | 0.87 (3) |
| K | 0.4948 | 0.5377 | -0.0154 | 4.34 (25) |
| $\mathrm{P}(1)$ | 0.1372 (21) | 0.6348 (12) | 0.3768 (20) | $1 \cdot 12$ (4) |
| $P\left(1^{i}\right)$ | -0.1664 (21) | 0.3606 (12) | -0.3812 (21) | 1.12 (4) |
| $\mathrm{P}(2)$ | 0.4926 (20) | $-0.0954(12)$ | 0.2103 (20) | 0.75 (3) |
| $P\left(2^{\prime}\right)$ | -0.5014 (20) | 0.0990 (12) | -0.2123 (20) | 0.75 (3) |
| O(11) | 0.0944 (22) | 0.5954 (11) | $0 \cdot 1071$ (20) | 1.93 (4) |
| $\mathrm{O}\left(11^{1}\right)$ | $-0.1148(22)$ | 0.4055 (11) | -0.1127 (20) | 1.93 (4) |
| $\mathrm{O}(12)$ | $0 \cdot 1460$ (20) | 0.7671 (11) | 0.4505 (20) | 1.03 (4) |
| $\mathrm{O}\left(12^{1}\right)$ | -0.1335 (20) | 0.2372 (11) | $-0.4209(20)$ | 1.03 (4) |
| O(13) | 0.4149 (21) | 0.5951 (11) | 0.5394 (22) | 2.24 (4) |
| $\mathrm{O}\left(13{ }^{\text {i }}\right.$ ) | -0.4285 (21) | 0.3986 (11) | 0.4605 (22) | $2 \cdot 24$ (4) |
| $\mathrm{O}(14)$ | -0.0652 (21) | $0 \cdot 5852$ (12) | 0.4108 (23) | $2 \cdot 15$ (4) |
| $\mathrm{O}\left(14^{\prime}\right)$ | 0.0576 (21) | 0.4146 (12) | -0.4159 (23) | $2 \cdot 15$ (4) |
| $\mathrm{O}(21)$ | 0.2406 (20) | -0.0388 (11) | 0.1432 (20) | 1.16(3) |
| $\mathrm{O}\left(21^{\prime}\right)$ | -0.2483 (20) | 0.0317 (11) | -0.1418 (21) | $1 \cdot 16$ (3) |
| $\mathrm{O}(22)$ | 0.7443 (19) | -0.0314 (11) | 0.3215 (20) | 1.18 (3) |
| $\mathrm{O}\left(22^{\prime}\right)$ | 0.2516 (20) | 0.0325 (11) | -0.3178 (20) | 1.18 (3) |
| $\mathrm{O}(23)$ | 0.5312 (20) | -0.1668 (12) | 0.3912 (20) | 1.27 (4) |
| $\mathrm{O}\left(23{ }^{1}\right)$ | 0.4682 (20) | $0 \cdot 1705$ (12) | -0.4055 (20) | 1.27 (4) |
| $\mathrm{O}(24)$ | 0.4623 (20) | $-0.1836(12)$ | $-0.0305(20)$ | 1.74 (4) |
| $\mathrm{O}(24)^{\prime}$ | $-0.4666(21)$ | 0.1831 (12) | 0.0342 (20) | 1.74 (4) |
| $\mathrm{O}(3)$ | 0.1428 (21) | 0.2463 (11) | 0.1761 (21) | 1.48 (4) |
| $\mathrm{O}\left(3^{1}\right)$ | $-0.1437(21)$ | $-0.2507(11)$ | -0.1728 (21) | 1.48 (4) |
| H(23) | 0.366 (3) | -0.1881 (14) | 0.405 (3) | 2.09 (6) |
| H(23) | -0.366 (3) | 0.1913 (14) | -0.418 (3) | 2.09 (6) |
| H(24) | 0.625 (3) | -0.1973 (14) | -0.064 (3) | 2.58 (8) |
| H(24) | 0.385 (3) | $0 \cdot 2058$ (15) | 0.074 (3) | $2 \cdot 58$ (8) |
| H(31) | 0.025 (3) | 0.2966 (16) | 0.063 (3) | 3.09 (7) |
| H(31) | -0.039 (3) | -0.2909 (16) | -0.063 (3) | 3.09 (7) |
| H(32) | 0.238 (4) | 0.3048 (16) | 0.314 (3) | 3.96 (11) |
| H(32) | -0.262 (4) | -0.2992 (16) | -0.315 (3) | 3.96 (11) |
| H(11) | -0.034 (3) | 0.4928 (21) | -0.015 (4) | 3.63 (19) |
| H(13) | 0.469 (3) | 0.5106 (17) | 0.509 (3) | 3.23 (22) |
| H(14) | 0.003 (4) | 0.5037 (25) | 0.509 (4) | 3.80 (14) |

Table 5. Selected bond distances in $\mathrm{Ca}_{2} \mathrm{KH}_{7}\left(\mathrm{PO}_{4}\right)_{4}$ $2 \mathrm{H}_{2} \mathrm{O}(\AA)$
E.s.d.'s for the least significant digits are given in parentheses.

|  | X-ray | Neutron |  |
| :--- | :--- | :--- | :--- |
|  |  | $(1)$ | $(2)$ |
| $\mathrm{Ca}-\mathrm{O}(12)$ | $2.386(1)$ | $2.424(10)$ | $2.354(10)$ |
| $\mathrm{Ca}-\mathrm{O}(21)$ | $2.344(1)$ | $2.370(11)$ | $2.317(11)$ |
| $\mathrm{Ca}-\mathrm{O}\left(21^{\prime}\right)$ | $2.473(1)$ | $2.485(9)$ | $2.459(10)$ |
| $\mathrm{Ca}-\mathrm{O}(22)$ | $2.315(1)$ | $2.310(9)$ | $2.315(9)$ |
| $\mathrm{Ca}-\mathrm{O}\left(22^{\prime}\right)$ | $2.513(1)$ | $2.507(9)$ | $2.509(9)$ |
| $\mathrm{Ca}-\mathrm{O}(23)$ | $2.608(1)$ | $2.611(10)$ | $2.608(9)$ |
| $\mathrm{Ca}-\mathrm{O}(24)$ | $2.704(2)$ | $2.707(10)$ | $2.717(10)$ |
| $\mathrm{Ca}-\mathrm{O}(3)$ | $2.471(1)$ | $2.462(10)$ | $2.502(10)$ |
| $\mathrm{K}-\mathrm{O}(11)$ | $2.635(3)$ | $2.676(11)$ |  |
| $\mathrm{K}-\mathrm{O}\left(11^{\prime}\right)$ | $2.838(2)$ | $2.801(11)$ |  |
| $\mathrm{K}-\mathrm{O}(13)$ | $2.931(2)$ | $2.891(12)$ |  |
| $\mathrm{K}-\mathrm{O}(14)$ | $2.765(2)$ | $2.757(12)$ |  |
| $\mathrm{K}-\mathrm{O}\left(14^{\prime}\right)$ | $2.881(2)$ | $2.891(13)$ |  |
| $\mathrm{P}(1)-\mathrm{O}(11)$ | $1.553(1)$ | $1.564(9)$ | $1.549(9)$ |
| $\mathrm{P}(1)-\mathrm{O}(12)$ | $1.514(1)$ | $1.567(9)$ | $1.472(9)$ |
| $\mathrm{P}(1)-\mathrm{O}(13)$ | $1.539(2)$ | $1.575(9)$ | $1.499(9)$ |
| $\mathrm{P}(1)-\mathrm{O}(14)$ | $1.535(2)$ | $1.464(9)$ | $1.604(9)$ |
| $\mathrm{P}(2)-\mathrm{O}(21)$ | $1.497(1)$ | $1.461(9)$ | $1.528(9)$ |
| $\mathrm{P}(2)-\mathrm{O}(22)$ | $1.498(1)$ | $1.499(8)$ | $1.496(9)$ |
| $\mathrm{P}(2)-\mathrm{O}(23)$ | $1.565(1)$ | $1.529(9)$ | $1.606(9)$ |
| $\mathrm{P}(2)-\mathrm{O}(24)$ | $1.581(1)$ | $1.583(10)$ | $1.583(10)$ |

Table 6. Geometry of the hydrogen bonds in $\mathrm{Ca}_{2} \mathrm{KH}_{7}-$ $\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$
E.s.d.'s for the least significant digits are given in parentheses.

| $\mathrm{O}-\mathrm{H} \cdots \mathrm{O}$ |  | $\mathrm{O}-\mathrm{H}(\AA)$ | $\mathrm{H} \cdots \mathrm{O}(\AA)$ | $\mathrm{O} \cdots \mathrm{O}(\AA) \angle \mathrm{O}-\mathrm{H} \cdots \mathrm{O}\left({ }^{\circ}\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| From the X-ray refinement |  |  |  |  |
| $\mathrm{O}(3)-\mathrm{H}(31) \cdots \mathrm{O}(11)$ | $0.94(2)$ | $1.91(2)$ | $2.801(2)$ | $157.9(23)$ |
| $\mathrm{O}(3)-\mathrm{H}(32) \cdots \mathrm{O}(13)$ | $0.95(4)$ | $2.01(4)$ | $2.892(2)$ | $154.6(32)$ |
| $\mathrm{O}(23)-\mathrm{H}(23) \cdots \mathrm{O}(12)$ | $0.85(3)$ | $1.71(3)$ | $2.563(2)$ | $171.6(26)$ |
| $\mathrm{O}(24)-\mathrm{H}(24) \cdots \mathrm{O}(3)$ | $0.71(4)$ | $2.02(4)$ | $2.720(2)$ | $169.7(27)$ |
| $\mathrm{O}(11)-\mathrm{H}(11) \cdots \mathrm{O}(11)$ | 1.26 | 1.26 | $2.514(2)$ | 180 |
| $\mathrm{O}(13)-\mathrm{H}(13) \cdots \mathrm{O}(13)$ | 1.24 | 1.24 | $2.483(2)$ | 180 |
| $\mathrm{O}(14)-\mathrm{H}(14) \cdots \mathrm{O}(14)$ | 1.24 | 1.24 | $2.486(2)$ | 180 |
| From the neutron refinement |  |  |  |  |
| $\mathrm{O}(3)-\mathrm{H}(31) \cdots \mathrm{O}\left(11^{\prime}\right)$ | $1.027(15)$ | $1.839(15)$ | $2.848(10)$ | $166.7(12)$ |
| $\mathrm{O}(3)-\mathrm{H}(32) \cdots \mathrm{O}\left(13^{\prime}\right)$ | $0.967(18)$ | $2.045(19)$ | $2.858(10)$ | $140.6(17)$ |
| $\mathrm{O}\left(3^{\prime}\right)-\mathrm{H}\left(31^{\prime}\right) \cdots \mathrm{O}(11)$ | $0.912(15)$ | $1.861(15)$ | $2.746(10)$ | $162.8(14)$ |
| $\mathrm{O}\left(3^{\prime}\right)-\mathrm{H}\left(32^{\prime}\right) \cdots \mathrm{O}(13)$ | $0.957(18)$ | $2.081(19)$ | $2.920(10)$ | $145.5(17)$ |
| $\mathrm{O}(23)-\mathrm{H}(23) \cdots \mathrm{O}(12)$ | $1.040(12)$ | $1.569(12)$ | $2.601(8)$ | $171.2(11)$ |
| $\mathrm{O}(24)-\mathrm{H}(24) \cdots \mathrm{O}\left(3^{\prime}\right)$ | $1.023(14)$ | $1.727(14)$ | $2.729(10)$ | $165.3(12)$ |
| $\mathrm{O}\left(23^{\prime}\right)-\mathrm{H}\left(23^{\prime}\right) \cdots \mathrm{O}\left(12^{\prime}\right)$ | $1.039(12)$ | $1.492(12)$ | $2.525(8)$ | $172.2(13)$ |
| $\mathrm{O}\left(24^{\prime}\right)-\mathrm{H}(24) \cdots \mathrm{O}(3)$ | $0.972(14)$ | $1.736(14)$ | $2.704(10)$ | $173.9(13)$ |
| $\mathrm{O}(11)-\mathrm{H}(11) \cdots \mathrm{O}\left(11^{\prime}\right)$ | $1.388(20)$ | $1.126(21)$ | $2.509(4)$ | $173.3(16)$ |
| $\mathrm{O}(13)-\mathrm{H}(13) \cdots \mathrm{O}\left(13^{\prime}\right)$ | $1.041(17)$ | $1.447(17)$ | $2.482(4)$ | $171.9(14)$ |
| $\mathrm{O}(14)-\mathrm{H}(14) \cdots \mathrm{O}\left(14^{\prime}\right)$ | $1.241(30)$ | $1.250(30)$ | $2.487(4)$ | $173.4(19)$ |



Fig. 1. Stereoscopic pair (Johnson, 1965) showing the unit cell of $\mathrm{Ca}_{2} \mathrm{KH}_{7}\left(\mathrm{PO}_{4}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$.
indication that the potassium does not occupy a single potential well located at the center of symmetry. Three hydrogen atoms, $\mathrm{H}(11), \mathrm{H}(13)$, and $\mathrm{H}(14)$, occupy centers of symmetry in model 2 but not in model 3 . As was expected, in view of the long $\mathrm{O}-\mathrm{O}$ distance, two of these hydrogen bonds refined to asymmetric configurations. $\mathrm{H}(14)$, however, remained very close to the center of the bond, although one of the oxygen atoms became unusually far away from its associated phosphorus atom. To determine whether this might be due to a false minimum, the hydrogen atom was displaced to a position $1.0 \AA$ away from the oxygen with the long $\mathrm{P}-\mathrm{O}$ distance, on the assumption that this oxygen was more likely to be the donor, and the refinement was repeated. In two cycles no other atom moved significantly, but $\mathbf{H}(14)$ returned to its position close to the center of the bond.


Fig. 2. The three hydrogen bonds that cross pseudo centers of symmetry. $\mathrm{O}(14)-\mathrm{H}(14)-\mathrm{O}\left(14^{1}\right)$ is still nearly symmetric in $P 1$. All of the hydrogen atoms have large amplitudes of thermal motion parallel to the bond.

Fig. 2 shows the three hydrogen bonds involving the hydrogen atoms that occupy centers of symmetry in models 1 and 2. It is apparent that all three of these hydrogen atoms have large amplitudes of thermal vibration nearly parallel to the $\mathrm{O}-\mathrm{O}$ vector, which implies that the potential wells, whether they have single or double minima, are rather flat in that direction.

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# Trithallium Tetraselenophosphate, $\mathrm{Tl}_{3} \mathrm{PSe}_{4}$, and Trithallium Tetrathioarsenate, $\mathrm{Tl}_{3} \mathbf{A s S}_{4}$, by Neutron Time-of-Flight Diffraction 

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#### Abstract

Room-temperature ( 293 K ) single-crystal structure determinations of the isostructural materials $\mathrm{Tl}_{3} \mathrm{PSe}_{4}$ and $\mathrm{Tl}_{3} \mathrm{AsS}_{4}$ were performed at the Los Alamos National Laboratory Pulsed Neutron Facility. For $\mathrm{Tl}_{3} \mathrm{PSe}_{4}: \quad M_{r}=959.92, \quad P c m n, \quad a=9.276(1), \quad b=$ $11.036(2), \quad c=9.058(1) \AA, \quad V=927.27 \AA^{3}, \quad Z=4$, $D_{m}=6.87(2), \quad D_{x}=6.876 \mathrm{Mg} \mathrm{m}^{-3}, \quad \lambda_{\text {neutron }}=$ $0 \cdot 5 \rightarrow 5 \cdot 2 \AA, F(000)=252 \cdot 5 \mathrm{fm}$. For $\mathrm{Tl}_{3} \mathrm{AsS}_{4}: M_{r}=$


816.29, Pcmn, $\quad a=9.084$ (3), $\quad b=10.877$ (3), $\quad c=$ $8.877(3) \AA, \quad V=877.11 \AA^{3}, \quad Z=4, \quad D_{m}=6.18$ (2), $D_{x}=6.181 \mathrm{Mg} \mathrm{m}^{-3}, \quad \lambda_{\text {neutron }}=0.5 \rightarrow 5.2 \AA, \quad F(000)=$ $177 \cdot 2 \mathrm{fm}$. For $\mathrm{Tl}_{3} \mathrm{PSe}_{4}\left(\mathrm{Tl}_{3} \mathrm{AsS}_{4}\right), 1929$ (1013) reflections were measured with $I>3 \sigma(I)$ and refined by full-matrix least squares to $R(F)=0.061(0.063)$. Results on atomic refinement from this study represent an order of magnitude increase in precision over
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[^0]:    * Lists of structure factors, anisotropic temperature factors and $\mathrm{O}-\mathrm{P}-\mathrm{O}$ angles have been deposited with the British Library Lending Division as Supplementary Publication No. SUP 39453 ( 38 pp.). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 5 Abbey Square, Chester CH1 2HU, England.

